Numerical Study of Non-Darcy Mixed Convection in a Vertical Porous Channel

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Nomenclature

C = inertia coefficient
Da = Darcy number
Gr^* = modified Grashof number, $(g\beta\Delta TKW)/\nu^2$
g = acceleration of gravity
h = heat transfer coefficient
K = permeability of the porous medium
k = thermal conductivity, W/m-K
Nu = local Nusselt number
Pe = Peclet number
Pr = Prandtl number
Re = Reynolds number based on channel width
T = temperature
U = dimensionless velocity in x direction
V = dimensionless velocity in y direction
$\mathbf{V}$ = velocity vector
V_0 = uniform inlet velocity
W = channel width
X = dimensionless distance in x direction
Y = dimensionless distance in y direction
β = thermal expansion coefficient of the fluid
ε = porosity
ζ = vorticity, $\partial V/\partial X - \partial U/\partial Y$
θ = dimensionless temperature, $(T - T_c)/(T_h - T_c)$
ν = kinematic viscosity
ψ = stream function

Introduction

RECENTLY, increasing efforts have been devoted to fundamental studies of fluid flow and heat transfer in fluid-saturated porous media. This interest is due to the importance of porous media in diverse engineering applications, including geothermal systems, building thermal insulation, enhanced oil recovery methods, nuclear waste disposal, packed-bed chemical reactors, and solid-matrix heat exchangers. Recent comprehensive reviews of mixed convection in porous media were reported by Kakac et al.[1] and Nield and Bejan.[2]

Relatively few investigations have been reported on mixed convection in vertical porous layers. Numerical and experimental studies of mixed convection in vertical porous annuli subjected to various boundary conditions were conducted by Parang and Keyhani,[3] Reda,[4] and Choi et al.,[5] among others. Limiting their study to Darcy flow regime, Lai et al.[6] reported numerical results of mixed convection in a vertical porous layer with a finite wall heat source.

In the present study, a numerical investigation of mixed convection in a vertical channel filled with a fluid-saturated porous medium is performed using the Brinkman-Forchheimer-extended Darcy model, which accounts for inertia effects as well as viscous effects at the boundary. The channel walls are maintained at a uniform temperature which is higher than the uniform inlet temperature of the fluid, such that buoyancy effects are assisting the "upward" flow. Detailed results of the evolution of mixed convection in the entrance region are reported for both Darcy and non-Darcy regimes.

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Analysis

It is assumed that the flow in the channel is steady, incompressible, and two-dimensional. The porous medium is considered to be homogeneous and isotropic, and is saturated with a single phase fluid which is in thermal equilibrium with the solid matrix. The thermophysical properties of the solid matrix and fluid are assumed to be constant except in the body force term of the momentum equations. Thermal dispersion in the porous matrix is assumed to be constant and it is incorporated in the effective thermal conductivity. The describing conservation equations of mass, momentum, and energy are converted into a vorticity-stream function formulation in the usual way, and the resulting dimensionless governing equations are

$$\nabla^2\psi = \zeta \tag{1}$$

$$\frac{1}{\varepsilon^2}\left(-\frac{\partial\psi}{\partial Y}\frac{\partial\zeta}{\partial X} + \frac{\partial\psi}{\partial X}\frac{\partial\zeta}{\partial Y}\right) = \frac{Gr^*}{Re^2 Da}\frac{\partial\theta}{\partial X} - \frac{1}{ReDa}\zeta - \frac{C}{\sqrt{Da}}\left(\frac{\partial}{\partial X}|\mathbf{V}|V - \frac{\partial}{\partial Y}|\mathbf{V}|U\right) + \frac{1}{Re}\nabla^2\zeta \tag{2}$$

$$-\frac{\partial\psi}{\partial Y}\frac{\partial\theta}{\partial X} + \frac{\partial\psi}{\partial X}\frac{\partial\theta}{\partial Y} = \frac{1}{Pe}\nabla^2\theta \tag{3}$$

The boundary conditions are such that no slip occurs at the impermeable walls. The flow at the inlet has a uniform "upward" velocity of V_0 and a uniform temperature T_c. The vertical walls are maintained at a uniform temperature T_h which is higher than the fluid inlet temperature. The boundary conditions at the exit were determined by the fully developed conditions.[7] The extent of the computational domain in the vertical Y direction is chosen to be large enough in order to achieve fully developed flow at the exit. This is accomplished by performing several trial runs for various combinations of Da, Re, and Gr^*. Selection of the proper channel length is made when the results in the entrance region do not change significantly with any further increase in channel length.

The heat transfer results are presented in terms of the local Nusselt number which is defined as

$$Nu = \left(-\frac{2}{\theta_W - \theta_m}\frac{\partial\theta}{\partial X}\right)_W \tag{4}$$

where θ_w and θ_m are the dimensionless wall and mean temperatures, respectively.

Numerical Procedure

Equations (1–3) and the corresponding boundary conditions were transformed into algebraic, finite difference equations using the control volume formulation outlined by Gosman et al.[8] The method ensures that the conservation laws are obeyed over each control volume, and it has been widely used to solve two-dimensional elliptic problems. The mesh size used in this analysis varied with the Darcy number. A skewed mesh was used along both coordinates. When the buoyancy effects were significant, a fine grid was used at the channel entrance and a progressively coarser grid was used further downstream. Similarly, along the X direction, the mesh was finer near the walls and coarser in the core region. In general, a 41×161 mesh was used except when buoyancy effects were small, in which case a 21×161 mesh provided sufficient accuracy.

The accuracy of the numerical model was verified by comparing the present results with results reported in the literature for several cases, including forced convection in a porous channel,[9] buoyancy-assisted flow in a vertical nonporous channel,[10] and mixed convection in a vertical porous layer with a finite wall heat source.[6] Very good agreement was found in all cases.

Results and Discussion

All the results were obtained with Pr = 0.70, and C in Forchheimer's extension of the momentum equations was fixed at 0.55.[2] In order to analyze the development of mixed convection in the vertical channel, the following four effects must be considered: 1) buoyancy, 2) bulk damping resistance due to the porous matrix, 3) inertial resistance in the porous medium, and 4) viscous effects at the boundary. The mixed convection flow in the porous channel can be classified as either Darcian or non-Darcian. The Darcy flow regime occurs when the inertia effects are negligible. The criteria which characterize each flow regime depend on the describing parameters Da, Gr^*, and Re. However, it was found that when the Darcy number was greater than 10^{-4} and the Reynolds number was greater than 1, the inertia effects were always significant.

Velocity and Temperature Profiles

The effects of Darcy number on the velocity distribution are shown in Fig. 1. In the nonporous channel, the flow is dominated by forced convection, the buoyancy effects are not appreciable enough to cause any flow distortion, and it is known in this case that the velocity distribution develops gradually from a flat profile at the inlet to a fully developed parabolic profile at the channel exit. In a porous channel, as the Darcy number is decreased while the modified Grashof number is kept constant, the buoyancy effects increase and the viscous effects are confined to the region near the walls. This results in increased distortions in the flow in the inlet region, and high velocity gradients near the walls. These distortions produce maximum velocities close to the wall region and minimum velocities in the core region in order to satisfy continuity (Fig. 1a). The profile becomes predominantly flat in the core region when the flow is fully developed. As the Darcy number is decreased further, the distortions in the flow increase, leading to smaller centerline velocity (Fig. 1b), and the maximum velocity increases and moves closer to the walls. In the Darcy flow regime ($Da < 10^{-4}$), both buoyancy and viscous effects are confined to a thin layer close to the walls where large velocities are produced while the core region is virtually unaffected.

The hydrodynamic development length increases as the Darcy number decreases (Fig. 1b), because the rate of increase of buoyancy is higher than the rate of increase of the porous medium resistance. However, the rate of increase of the hydrodynamic entry length becomes smaller at lower Darcy numbers due to the increased bulk damping.

In the entrance region the temperature profile exhibits boundary-layer behavior at low Darcy number, with a large gradient near the walls and zero gradient in the core region. However, a quick development of the profile occurs as the fluid progresses into the channel, especially at lower Da values. The axial variation of the bulk (mean) temperature is presented in Fig. 2. Due to heating from the walls, the bulk temperature increases in the entrance region and approaches the asymptotic value of 1 at fully developed conditions. As the Darcy number decreases, the slope of the bulk temperature variation in the inlet region is steeper due to the resulting increase in buoyancy effects which tend to equalize the temperature in the fluid. Thus, as the Darcy number decreases, the thermal development length decreases significantly.

Heat Transfer Results

The information presented so far on velocity and temperature distributions suggests that a significant increase in heat transfer occurs in the entrance region as the Darcy number decreases, as shown in Fig. 3. Very close to the channel inlet, the effect of decreasing Darcy number is important only at lower values of Da (Darcy regime). At higher Da values, the flow in this region is dominated by forced convection and the Nusselt number remains constant with changing Da. The mixed convection region, which starts a short distance from the inlet, exhibits a significant increase in heat transfer with decreasing Darcy number. In the fully developed region, the asymptotic

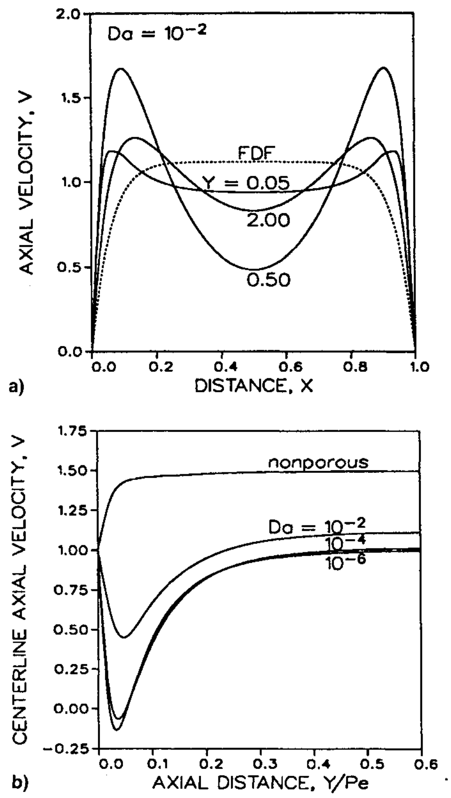

Fig. 1 Effects of Darcy number on the velocity distribution for Gr^* = 250, Re = 20: a) velocity distribution on $Da = 10^{-2}$ and b) streamwise centerline velocity.

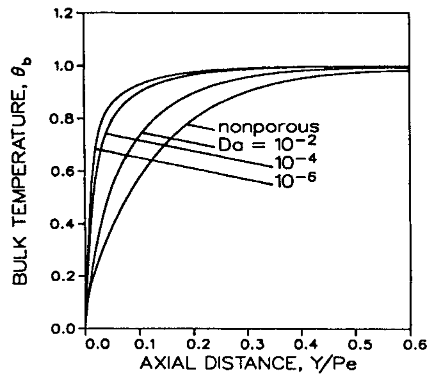

Fig. 2 Effects of Darcy number on the axial bulk temperature variation for Gr^* = 250, Re = 20.

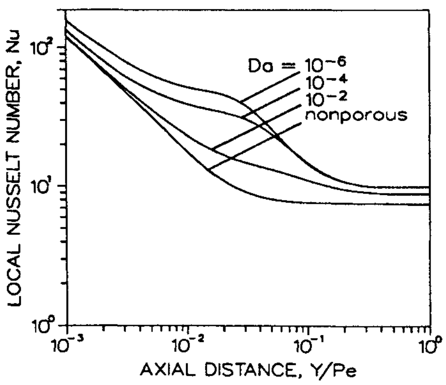

Fig. 3 Effects of Darcy number on the local Nusselt number variation for Gr^* = 250, Re = 20.

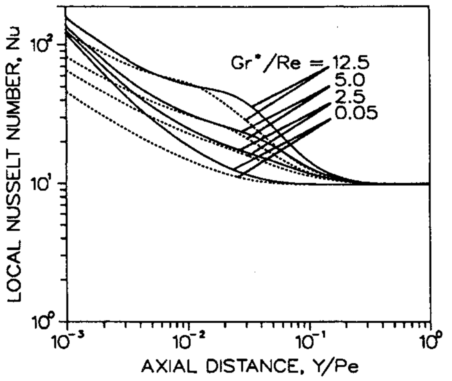

Fig. 4 Effects of Gr^*/Re on the local Nusselt number variation $Da = 0$–10^{-6}, $Re = 20$ (——), and $Re = 100$ (- - - - -).

value of the Nusselt number increases only at higher Darcy number and remains virtually the same at lower Darcy number.

In the Darcy flow regime it was found that for constant Prandtl number and sufficiently high Reynolds number, heat transfer is governed solely by Gr^*/Re ratio (Fig. 4) instead of Gr^*/Re^2 ratio which is commonly used for mixed convection in nonporous media. This can be verified by an order of magnitude analysis of Eq. (2). In Fig. 4, it is shown that in the inlet and mixed convection region, the Nusselt number increases with increasing Gr^*/Re, and in the fully developed region all the curves converge to the same asymptotic value. However, at moderately low Reynolds number, the axial conduction effects were found to be more significant than in the nonporous channel and Gr^*/Re is no longer the governing parameter as illustrated in Fig. 4.

Conclusions

A numerical study of buoyancy-assisted mixed convection in a vertical porous channel maintained at uniform wall temperatures is performed using the Brinkman-Forchheimer-extended Darcy model.

The results show that when the Darcy number is decreased while the modified Grashof number and the Reynolds number are kept constant, there is a significant increase of the velocities near the walls leading to increased heat transfer, especially in the mixed convection region which starts closer to the channel inlet than in the nonporous channel. In the Darcy flow regime, the governing parameter is Gr^*/Re, but at moderately low Reynolds number, the axial conduction effects (Peclet number effects) are significant.

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Combined Surface Radiation and Free Convection in Cavities

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Nomenclature

A = aspect ratio, H/d
d = spacing, m
F_{ij} = view factor from the ith element to the jth element
g = acceleration due to gravity, m/s^2
H = height of the enclosure, m
J_i = elemental radiosity, W/m^2
j_i = elemental dimensionless radiosity, $J_i/\sigma T_H^4$
k = thermal conductivity of fluid, W/m K
N_{RC} = radiation conduction interaction parameter, $\sigma T_H^4 d/[(T_H - T_c)k]$
Nu_c = local convection Nusselt number, $-(\partial\phi/\partial Y)_{y=0}$
$\overline{Nu_c}$ = mean convection Nusselt number, $\int_0^{2A} Nu_c \, dX/2A$
$\overline{Nu_o}$ = mean overall Nusselt number defined in Eq. (6)
Nu_R = radiation Nusselt number, $[q_{\rm rad} d/k(T_H - T_c)N_{RC}]$
$\overline{Nu_R}$ = mean radiation Nusselt number, $\int_0^{2A} Nu_R \, dX/2A$
Pr = Prandtl number, ν/α
$q_{\rm rad}$ = elemental radiative heat flux based on d, $\varepsilon/(1 - \varepsilon)(\sigma T_i^4 - J_i)$ W/m^2 ($\varepsilon \neq 1$, ε of all elements on a wall is same)
Ra = Rayleigh number based on d, $g\beta(T_H - T_c)d^3/\nu\alpha$
T = temperature, K
T_c = temperature of the right wall of the enclosure, K
T_H = temperature of the left wall of the enclosure, K
T_i = temperature of the ith element on the wall
T_R = temperature ratio, T_c/T_H
U = dimensionless vertical velocity, ud/α
u = vertical velocity, m/s
V = dimensionless horizontal velocity, vd/α
v = horizontal or cross velocity, m/s
W = dimensionless vorticity, wd^2/ν
X = dimensionless vertical coordinate, x/d
x = vertical coordinate, m
Y = dimensionless horizontal coordinate, y/d
y = horizontal coordinate, m
α = thermal diffusivity of fluid, m^2/s

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